Title Environmental Policy, Health, and Growth[1]

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Abstract

This paper develops a standard neoclassical model of growth in which pollution affects individuals' health and the government can influence the quality of the environment via a tax on emissions. In such an economy, we analyze the effects of a change in this policy on the trade-offs between the resources allocated to abatement, health and consumption (or savings). We demonstrate that less pollution lowers healthcare spending, and show the existence of an inverted U-shaped relationship between the pollution tax and (i) the level of health; (ii) consumption; and (iii) welfare. Results are analyzed both at steady state and along the transition path.

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1. Introduction

It is well established that pollution emissions which deteriorate the quality of the environment also affects individuals' health. For instance, in 2007, the World Health Organization stated that 24% of global disease burden and 23% of all deaths could be attributed to environment factors. In particular, 42% of chronic obstructive pulmonary disease could be attributed to occupational exposures to dust and chemicals, as well as indoor air pollution from household solid fuel use.

Studying the relationship between pollution, economic development and health is important not only because health is a factor of production (affecting the productivity of individuals) but also because it affects individuals' welfare (see e.g. the seminal article by Grossman, 1972). In the standard environmental literature in which we can include Gradus and Smulders (1993), Brock and Taylor (2005) and Xepapadeas (2005), however, this issue has been overlooked.[2] More precisely, the authors do not explicitly take into account the costly aspect of health and thus the resulting trade-offs between health, capital investment, and individuals' consumption.

In the present paper, in contrast, we explicitly formulize a health sector to analyze these potential trade-offs. We should mention that Grossman (1972), van Zon and Muysken (2001) and Aísa and Pueyo (2006) also analyzed some of these trade-offs, but they did not discuss those related to pollution. In that sense, this paper builds on two strands of the literature in a unified framework: that analyzing the relation between growth and the environment; and that studying the issue of growth and health.

Another contribution of this paper is the analysis of the short-run dynamic of individuals' behaviors in respect to healthcare spending, consumption and savings when the level of pollution varies. This issue, surprisingly, has also been overlooked in most of the articles cited above.

To conduct the analysis and analyze how a pollution tax affects individuals' behaviors and economic development, we develop a Ramsey-Cass-Koopman model. Although the model is standard, yet it is rich enough to capture a number of important features. Note that, this paper does not analyze the growth rates in the steady state. That is, developing the Ramsey-Cass-Koopman model, which predicts an exogenous growth rate in the steady state, is probably fit with the objectives of this paper. As stated by Barro and Sala-i-Martin (2004), the key element of this model is that it provides a useful

[2] See Rici (2007) for a recent comprehensive survey in the environmental literature.

benchmark to analyze individuals' behaviors. Moreover, it helps us to discuss about welfare issues in a clear-cut manner.

In our model, firms produce an output which is polluting. Pollution emissions (by product of production) affect the level of health (productivity) of individuals and their welfare. Government can reduce emissions by using a pollution tax whose proceeds are invested in abatement technologies. Thereby, the aim of this paper is to analyze the effects of a change in the pollution tax both in the long run (steady state) and in the short run (along the transition).

Our main findings can be summarized as follows. First, we demonstrate that an increase in abatement spending (a higher pollution tax) reduces healthcare spending both in the short run and in the long run. It means that the health benefit of an improved environmental quality allows people to spend less resource on healthcare services. Interestingly, this finding is supported by empirical data. For instance, Romley *et al.* (2010) showed that improved air quality have reduced total spending on hospital care by $193,100,184 in total in California over 2005-2007. In our model, this result is implicitly taken into account by the fact that environmental quality and healthcare spending are imperfect substitutes.

Second, we show the existence of three inverted U-shaped relationships resulting from the trade-offs discussed above: (i) the first one between the pollution tax and the level of health, (ii) the second one between the pollution tax and consumption, and (iii) the third one between the pollution tax and welfare. This result suggests that if the productivity of abatements is relatively high, a tighter environmental policy makes people better-off because this policy increases both the level of consumption and health. However, if abatements become less productive, people have to sacrifice consumption in exchange for a higher quality of the environment as well as a higher level of health. Herein, we have two possible outcomes on welfare: People are better-off if the welfare gain from a better health is greater than the welfare loss from a lower level of consumption; otherwise, people are worse-off. Finally, if the productivity of abatements is very low, a better environmental quality requires an amount of resources so that an increase in the pollution tax reduces the level of consumption, health and welfare.

Turning to the analysis of the short-run dynamics of the model, we show that individuals face an instantaneous loss in consumption at the time of the policy change. Then, along the transitional path, the outcome depends on the productivity of abatements. If it is high, an increase in environmental care leads to a great improvement in environmental quality, labor productivity and growth. That is, both consumption and physical capital increase along the transition. However, if the productivity of abatements is low, it crowds out the investments which are necessary to promote growth. As a result, consumption and physical capital decrease.

The remaining of this paper is organized as follows. Section 2 presents the model. In Section 3, we investigate the short-run and long-run effects of a pollution tax on individuals' behaviors and welfare, and then we construct some static comparative to examine the responses of the economy following a change in some structural parameters. Section 4 concludes.

2. Model

Consider a closed economy in continuous time. Time, denoted by t, goes from zero to infinity: $t \in [0, \infty)$. At each instant, there is a representative household owning K_t units of wealth and comprising L_t identical members growing at an exogenous rate, $n > 0$: $\dot{L}_t = nL_t$. Each individual has one unit of labor that is supplied inelastically to output production. Output, Y_t, can be consumed, C_t, spent on health services, Z_t, spent on abatements to reduce pollution, Q_t, or invested to give new units of physical capital, $\dot{K}_t$. For simplicity, we assume that depreciation of physical capital is zero. The resource constraint is then given by:

$$Y_t = C_t + Z_t + Q_t + \dot{K}_t \quad . \tag{1}$$

The technology of the output production is assumed to be given by:

$$Y_t = B\left(K_t\right)^{\alpha}\left(h_t A_t L_t\right)^{1-\alpha} \quad , \tag{2}$$

where $0 < \alpha < 1$, $B > 0$ is a constant productivity parameter, A_t is technical progress evolving at an exogenous rate of growth, $g_A > 0$ (i.e., $\dot{A}_t = g_A A_t$) and h_t is the level of health of an individual. A notable feature of the production function is that it clearly distinguishes between the standard efficient units of labor (i.e., the combination of raw labor and knowledge, $A_t L_t$, as in the text book of Barro and Sala-i-Martin, 2004) and healthy units of efficient labor, $h_t A_t L_t$.

The novelty of this paper is the introduction of a health production sector whereby individuals choose how much resource to spend on healthcare services. The noteworthy somewhat interesting feature of the health technology is that it is negatively affected by pollution emissions. For simplicity, following Aloi and Tournemaine (2010), we set:

$$h_t = \phi\left(\eta_t\right)^{\gamma}\left(P_t\right)^{-\chi} \quad , \tag{3}$$

where $\phi > 0$, $\gamma > 0$, $\chi > 0$, P_t denotes pollution emissions, $\eta_t \equiv Z_t / Y_t$ is the fraction of output devoted to healthcare services.[3] Note that in the above technology, we assume that healthcare spending are the percentage of output rather than its level. This formalization, which is taken from Aísa and Pueyo (2006), allows us to simplify the analysis and to avoid the problem of exploding growth paths.

To keep the analysis simple, we focus on the immediate effects of emissions, such as air pollution, whose implications on health are for the most part direct and are drastically reduced when addressed (see, e.g., Künzli, 2002). It should be noted, however, that none of the results we derive in this paper hinge on this assumption. In line with Gradus and Smulders (1993) and Brock and Taylor (2005), we assume that pollution is a by-product of output production. As mentioned, these emissions can be reduced through abatements that consume output; in so doing the flow of pollution does not grow without bound and is constant at steady-state. Formally, we have

$$P_t = \left(\frac{Y_t}{Q_t} \right)^{\beta} , \tag{4}$$

where $\beta > 0$.

Turning to the specification of preferences, in line with Grossman (1972), van Zon and Muysken (2001) and Aloi and Tournemaine (2010), among others, it is assumed that individuals derive utility from consumption and health:

$$U = \int_0^\infty \frac{\left[c_t \left(h_t \right)^{\theta} \right]^{1-\sigma} - 1}{1-\sigma} e^{-\rho t} dt , \tag{5}$$

where $\sigma > 0$ is the inverse of the elasticity of substitution, $\rho > 0$ is the rate of time preference, $\theta > 0$ measures the relative contribution of health to utility, and $c_t \equiv C_t / L_t$ represents per capita consumption.

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[3] Technology (3) can be rationalized by a technology for health displaying decreasing returns to scale, such as: $\dot{h}_t = \upsilon \left[\eta_t h_t \right]^{\xi} - \varsigma P_t h_t$, where $0 < \xi < 1$, $\upsilon > 0$, $\varsigma > 0$. The intuition behind this dynamic equation is that people must continuously use resources to keep healthy. Intuitively, pollution reduces the production of health and/or increases the depreciation rate of health. Because of decreasing returns, in the long-run the level of health is constant ($\dot{h}_t = 0$). I.e., with appropriate parameter values, the law of motion of health yields the technology captured by (3).

Before closing this section, it is convenient to rewrite the model in terms of variables per unit of efficient labor. This will allow us to simplify the computations and the analysis of both the steady state and the transitional dynamics. Defining $y_t \equiv Y_t / \left(A_t L_t\right)$, $k_t \equiv K_t / \left(A_t L_t\right)$ and $c_t \equiv C_t / \left(A_t L_t\right)$, the law of motion of physical capital becomes:

$$\dot{k}_t = \left(1-\eta_t-\mu_t\right) B\left(k_t\right)^{\alpha} \left[\phi\left(\eta_t\right)^{\gamma} \left(\mu\right)^{\beta\chi}\right]^{1-\alpha} - c_t - \left(g_A + n\right) k_t \ , \tag{6}$$

where $\mu_t \equiv Q_t / Y_t$ is the fraction of output devoted to abatements.

The output technology becomes

$$y_t = B\left(k_t\right)^{\alpha} \left(h_t\right)^{1-\alpha} \ . \tag{7}$$

Lastly, the preferences are rewritten as

$$U = \int_0^{\infty} \frac{\left[A_t c_t \left(h_t\right)^{\theta}\right]^{1-\sigma} - 1}{1-\sigma} e^{-\rho t} dt \ . \tag{8}$$

3. Equilibrium

In this section, we characterize the equilibrium of the model. For simplicity, we assume that abatements are public activities. These are funded by a pollution tax, τ, levied on output, so that $Q_t = \tau Y_t$ at each moment. As mentioned by Aloi and Tournemaine (2010), we can rationalize our approach by appealing to the fact that governments may actually promote the adoption of technologies that reduce pollution originating from the use of resources, such as coal or fuel, impairing air or water quality. For example, may promote: renewable energies to replace fossil fuel (Künzli, 2002), "green" buses for public transport, "green" power stations for energy, or water purification systems removing contaminants and other harmful micro organisms from rivers and water sources.

This remaining of this section is organized as follows. First, we characterize the first order conditions. We then analyze the solution of the model in steady state and along a transition. Finally, we construct some static comparative to examine the responses of the economy following a change in some structural parameters.

3.1. The first order conditions

The problem of the representative individual is to choose the level of consumption per unit of efficient labor, c_t, and the fraction of income (output) devoted to healthcare services, η_t, that maximize lifetime utility (8) subject to the law of motion of physical capital per unit of efficient labor (6) and the initial condition $k_0 > 0$. The Current-Value Hamiltonians to this problem is

$$CVH = \frac{\left\{A_t c_t \left[\phi(\eta_t)^{\gamma} (\tau)^{\beta\chi}\right]^{\theta}\right\}^{1-\sigma} - 1}{1-\sigma} + \lambda_t \left\{(1-\eta_t-\tau) B(k_t)^{\alpha} \left[\phi(\eta_t)^{\gamma} (\tau)^{\beta\chi}\right]^{1-\alpha} - c_t - (g_A + n) k_t\right\},$$

where λ_t is the co-state variable associated to the law of motion of capital and we have used $\mu_t = Q_t / Y_t = \tau$. The solution to this problem is defined by the first order conditions: $\partial CVH / \partial c_t = 0$; $\partial CVH / \partial \eta_t = 0$; $\partial CVH / \partial k_t = -\dot{\lambda}_t + \rho \lambda_t$; and the transversality condition: $\lim_{t\to\infty} \lambda_t k_t e^{-\rho t} = 0$.

After some manipulations, we get:

$$\frac{\left\{A_t c_t \left[\phi(\eta_t)^{\gamma} (\tau)^{\beta\chi}\right]^{\theta}\right\}^{1-\sigma}}{c_t} = \lambda_t , \tag{9}$$

$$\frac{\gamma\theta \left\{A_t c_t \left[\phi(\eta_t)^{\gamma} (\tau)^{\beta\chi}\right]^{\theta}\right\}^{1-\sigma}}{\eta_t} + \lambda_t \gamma (1-\alpha)(1-\eta_t-\tau) \frac{y_t}{\eta_t} = \lambda_t y_t , \tag{10}$$

$$\alpha(1-\eta_t-\tau)\frac{y_t}{k_t} - (g_A + n) + \frac{\dot{\lambda}_t}{\lambda_t} = \rho . \tag{11}$$

Expression (9) shows that the marginal utility of consumption equals the marginal cost of wealth. Equation (10) ensures that the marginal benefit of an additional fraction of output spent on healthcare services (measured by the utility gain plus the production gains) equals its marginal cost (measured by output losses). Expression (11) shows that the rate of return on wealth (capital) equals the discount rate. The rate of return on wealth is given by the marginal productivity of capital net of the marginal cost to maintain capital at its existing level (break-even investment) plus the change in the shadow price.

3.2. Steady State

After setting out the problem of the representative individual, we now focus on the steady state. In steady state, by construction, the growth rates of variables per unit of efficient labor are zero: $\dot{y}_t = \dot{k}_t = \dot{c}_t = \dot{h}_t = \dot{\eta}_t = 0$. After computations gathered in Appendix, we obtain:

$$\eta_{ss} = \frac{\gamma\left[\theta\left(\sigma g_A + \rho + n - \alpha g_A - \alpha n\right) + (1-\alpha)\left(\sigma g_A + \rho + n\right)\right](1-\tau)}{\left(1+\gamma-\gamma\alpha\right)\left(\sigma g_A + \rho + n\right) + \gamma\theta\left(\sigma g_A + \rho + n - \alpha g_A - \alpha n\right)} , \quad (12)$$

$$h_{ss} = \phi\left(\eta_{ss}\right)^{\gamma}(\tau)^{\beta\chi} , \quad (13)$$

$$k_{ss} = \left\{\frac{B\alpha\left(1-\eta_{ss}-\tau\right)\left[\phi\left(\eta_{ss}\right)^{\gamma}(\tau)^{\beta\chi}\right]^{1-\alpha}}{\sigma g_A + \rho + n}\right\}^{\frac{1}{1-\alpha}} , \quad (14)$$

$$c_{ss} = \frac{\sigma g_A + \rho + n - \alpha g_A - \alpha n}{\alpha} k_{ss} , \quad (15)$$

where the symbol "ss" is used to denote any variable in steady state.

In the remaining of this subsection, we analyze: (i) the influences of a tighter environmental policy on healthcare spending, η_{ss}, the level of health, h_{ss}, capital, k_{ss}, and consumption, c_{ss}; (ii) the effects of a tighter environmental policy on welfare.

3.2.1. Policy implication

We now study how a tighter environmental policy affects η_{ss}, h_{ss}, k_{ss}, and c_{ss}. Examination of equations (12)-(15) reveals that there exists a level of pollution tax, $\tau_{c,k}^{\max}$, that maximizes both of c_{ss} and k_{ss}, and another level, $\tau_{h}^{\max}$, that maximizes h_{ss}. In Appendix, we show:

$$\tau_{c,k}^{\max} = \frac{\beta\chi(1-\alpha)}{(\beta\chi+\gamma)(1-\alpha)+1} , \quad (16)$$

$$\tau_{h}^{\max} = \frac{\beta\chi}{\beta\chi+\gamma} , \quad (17)$$

where

$$\tau_{c,k}^{\max} < \tau_{h}^{\max} .$$

The following table shows the sign of the first-order derivative of η_{ss}, h_{ss}, k_{ss}, and c_{ss} with respect to τ.

Table 1: The effects of a tighter environmental policy on η_{ss}, h_{ss}, k_{ss}, and c_{ss}.

	$0 < \tau \le \tau_{c,k}^{\max}$	$\tau_{c,k}^{\max} < \tau \le \tau_{h}^{\max}$	$\tau_{h}^{\max} < \tau < 1$
$\partial \eta_{ss} / \partial \tau$	< 0	< 0	< 0
$\partial h_{ss} / \partial \tau$	> 0	≥ 0	< 0
$\partial k_{ss} / \partial \tau = \partial c_{ss} / \partial \tau$	≥ 0	< 0	< 0

Table 1 shows that $\partial \eta_{ss} / \partial \tau < 0$ for any level of pollution tax. This result means that a tighter environmental policy reduces the fraction of output devoted to healthcare services. In other words, the health benefit of improved environmental quality allows individuals to spend less resource on healthcare services.

An important result from Table 1 is that the direct in which a tighter environmental policy influences h_{ss}, k_{ss}, and c_{ss} crucially depends on its initial level. We will come back on this issue in the section on welfare. As a preamble, we can observe that for a low level of the pollution tax ($0 < \tau \le \tau_{c,k}^{\max}$), a tighter environmental policy has a positive effect on the level of health. This is because, in this case, the productivity of abatements is high. Interestingly, this effect carries on consumption and capital: that is, the health benefits of improved environmental quality allow individuals to save an amount of resources which can be used for consumption and capital accumulation. When the pollution-tax rate takes higher values ($\tau_{c,k}^{\max} < \tau \le \tau_{h}^{\max}$), the productivity of abatement is still sufficient to increase environmental quality which, in turn, leads to a higher level of health. However, as abatements become more costly, a tighter environmental policy now reduces consumption and capital accumulation.

Lastly, for high values of the pollution-tax rate ($\tau_{h}^{\max} < \tau < 1$), the productivity of abatements becomes so small that an increase in abatement spending would reduce the level of health, consumption and capital. That is, individuals must sacrifice their present and future consumption (savings) as well as their level of health to maintain the environmental quality as its steady level. The following proposition summarizes our main results.

Proposition 1 *On the long-run effects of a tighter environmental policy,* τ *:*

(a) At any level of environmental care, a tighter environmental policy reduces healthcare spending;

(b) If $0 < \tau \le \tau_{c,k}^{\max}$ *, a higher level of pollution tax increases both consumption and the level of health;*

(c) If $\tau_{c,k}^{\max} < \tau \leq \tau_{h}^{\max}$*, an increase in abatement spending improves the level of health but reduces consumption;*

(d) If $\tau_{h}^{\max} < \tau < 1$*, better environmental quality is associated with less consumption and a lower level of health.*

3.2.2. Welfare analysis

In this part, we characterize the effects of a tighter environmental policy on welfare. We assume that the economy is initially in steady state. After some manipulations, we obtain the following lifetime utility function:

$$U = \frac{\left[A_0 c_0 \phi^{\theta} \left(\eta_0\right)^{\gamma\theta} \left(\tau\right)^{\beta\chi\theta}\right]^{1-\sigma}}{(1-\sigma)\left[(\sigma-1)g_A+\rho\right]} \quad . \tag{18}$$

Straightforward manipulations of (18) allow us to establish the following:

Proposition 2: Define τ^{w} as the welfare-maximizing tax rate. Then, we have:

$$\tau^{w} = \frac{\beta\chi(1-\alpha)\left[\alpha+\theta\left(\sigma g_A+\rho+n-\alpha g_A-\alpha n\right)\right]}{(\beta\chi+\gamma)(1-\alpha)\left[\alpha+\theta\left(\sigma g_A+\rho+n-\alpha g_A-\alpha n\right)\right]+\alpha}$$

where:

$$\tau_{c,k}^{\max} < \tau^{w} < \tau_{h}^{\max} .$$

This Proposition is important because it allows us to refine our analysis of the effects of pollution tax on individuals' behaviors in respect to consumption, healthcare spending and savings. From this condition, we can study four main cases: (i) $\left(0;\tau_{c,k}^{\max}\right]$; (ii) $\left(\tau_{c,k}^{\max};\tau^{w}\right]$; (iii) $\left(\tau^{w};\tau_{h}^{\max}\right]$; and (iv) $\left(\tau_{h}^{\max};1\right)$. These are represented graphically in Figure 1 and analyzed in turn below.

Figure 1 here

Case 1: $0 < \tau \leq \tau_{c,k}^{\max}$

When the economy devotes a low fraction of output to abatements, a greater amount of abatement spending increases both consumption and the level of health in the long run (Proposition 1). Hence, people are better-off. In other words, this result implies that the government can increase welfare by shifting up the level of the pollution tax. This is known as double dividends of the environmental care.[4]

Case 2: $\tau_{c,k}^{\max} < \tau \le \tau^{w}$

When the pollution-tax rate takes a value in the set $\left(\tau_{c,k}^{\max}, \tau^{w}\right]$, a tighter environmental policy leads to a higher level of health and less consumption (Proposition 1). The welfare gain from the increase in the level of health is greater than the welfare loss due to the decrease in the level of consumption. Thus, this policy makes individuals better-off. Note that maximum welfare is attained if $\tau = \tau^{w}$.

Case 3: $\tau^{w} < \tau \le \tau_{h}^{\max}$

In this case, a greater amount of abatement spending improves the level of health of individuals but reduces consumption (Proposition 1). In contrast with Case 2, the welfare gain from the increase in the level of health is lower than the welfare loss due to the decrease in consumption. Consequently, individuals are worse-off.

Case 4: $\tau_{h}^{\max} < \tau < 1$

Here, a tighter environmental policy reduces both the level of health of individuals and consumption (Proposition 1). Hence, it makes people worse-off. By cutting down the pollution-tax rate, the government reduces tax burden on individuals, thus shifts up consumption, the level of health and welfare.

To summarize, as shown in Figure 1 and as depicted in Case 1-4, we obtain three inverted U-shaped relationships between the pollution tax and (i) the level of health; (ii) consumption; and (iii) welfare. As pollution tax increases from 0 to $\tau_{c,k}^{\max}$, consumption per unit of efficient labor increases to $c_{\max}$, the level of health is improved to h^{i}, and welfare rises to U^{i} (Case 1). Along with tax level from $\tau_{c,k}^{\max}$ to τ^{w}, while consumption decreases from $c_{\max}$ to c^{ii}, the level of health increases from h^{i} to h^{ii}, and welfare goes up from U^{i} to its maximum level, $U_{\max}$ (Case 2). As pollution tax increases from τ^{w} to 1, the level of consumption and welfare decline continuously. The level of health, however,

[4] In Smulders and Gradus (1996), double dividends mean that the benefits of environmental improvement extend to other fields. In Glomm (2004), the double dividends include an efficiency dividend (a higher consumption) and a green dividend (a better environmental quality).

increases from h^{ii} to its maximum level, h_{max}, when pollution tax rises from τ^{w} to $\tau_{h}^{\max}$ (Case 3); then it decreases when pollution tax increases from $\tau_{h}^{\max}$ to 1 (Case 4).

An issue arisen from the above discussion is the following: which situations are we likely to observe in the real world? In fact, Case 3 and Case 4 can be excluded because there is no reason to keep such level of pollution tax. For the other cases, however, we can assume that developed countries usually impose a tighter policy than do developing countries (see, e.g., Mukhopadhyay, 2006). Hence, it is possible to take the level of pollution tax as a proxy for the development level of a country. Accordingly, Case 1 would represent the developing countries while Case 2 would represent developed countries. That is, developing countries can increase both consumption and the level of health while developed countries face a trade-off between the two. To give evidence of this idea, we emphasize a finding in Jones and Klenow (2010). At first, we rewrite their Table 2 with a few rearrangements.[5]

Table 2: Per capita consumption and life expectancy across countries, 2000.

Country	Group	Life expectancy	Per capita consumption
Malawi *	1	46	0.028
Botswana	1	48.9	0.115
South Africa	1	56.1	0.186
India *	1	62.5	0.058
Russia	1	65.3	0.140
Indonesia *	1	67.5	0.087
Thailand *	1	68.3	0.125
Brazil *	1	70	0.188
China	1	71.4	0.079
Mexico *	1	74	0.194
South Korea	2	75.9	0.273
The United States *	2	77	0.762
The United Kindom *	2	77.7	0.551
Germany *	2	77.9	0.534

[5] To create our Table 2, we used column "Country", "Per capita income", "LifeExp" and "C/Y" in Table 2 of Jones and Klenow (2010).

Singapore	2	78.1	0.353
France *	2	78.9	0.505
Italy	2	79.5	0.473
Hong Kong	2	80.9	0.586
Japan *	2	81.1	0.476

In Table 2, we sort countries by ascending order of their life expectancy. We categorize these countries into two groups: Group 1 consists of ten developing countries; Group 2 consists of nine developed countries. Looking at the countries marked by symbol "*", we discover two points. First, the developing countries with higher per capita consumption also have longer life expectancy. Second, the developed countries with higher per capita consumption have shorter life expectancy. This discovery is compatible with our analysis in Case 1 and Case 2.

3.3. Transitional Dynamics

We now characterize the transitional dynamics of the model. As shown in Appendix, by log-linearizing and taking the first order of Taylor's expansion of equation (9), (10), and (11), we obtain the following system of three equations:

$$c_t = \frac{\left[1+\gamma(1-\alpha)\right]\eta_t - \gamma(1-\alpha)(1-\tau)}{\gamma\theta} B\left(k_t\right)^{\alpha}\left[\phi\left(\eta_t\right)^{\gamma}(\tau)^{\beta\chi}\right]^{1-\alpha}. \tag{19}$$

$$\frac{d\ln k_t}{dt} = D_{11}\left(\ln k_t - \ln k_{ss}\right) + D_{12}\left(\ln\eta_t - \ln\eta_{ss}\right)\ , \tag{20}$$

$$\frac{d\ln k_t}{dt} = D_{21}\left(\ln k_t - \ln k_{ss}\right) + D_{22}\left(\ln\eta_t - \ln\eta_{ss}\right)\ , \tag{21}$$

where

$$D_{11} = \left[\frac{1+\gamma(1-\alpha+\theta)}{\gamma\theta}\eta_{ss} - \frac{(1-\alpha+\theta)(1-\tau)}{\theta}\right](1-\alpha)\frac{y_{ss}}{k_{ss}}\ ,$$

$$D_{12} = \left[\frac{(1-\alpha+\theta)(1-\tau)\gamma(1-\alpha)}{\theta} - \frac{[1+\gamma(1-\alpha+\theta)][\gamma(1-\alpha)+1]}{\gamma\theta}\eta_{ss}\right]\frac{y_{ss}}{k_{ss}}\ ,$$

$$D_{21} = \frac{\left[\left(1-\sigma\frac{1+\gamma(1-\alpha+\theta)}{\gamma\theta}\right)\eta_{ss} - 1 + \tau + \sigma\frac{(1-\alpha+\theta)(1-\tau)}{\theta}\right]\alpha(1-\alpha)\frac{y_{ss}}{k_{ss}}}{\frac{\sigma[1+\gamma(1-\alpha)]\eta_{ss}}{[1+\gamma(1-\alpha)]\eta_{ss}-\gamma(1-\alpha)(1-\tau)} + \sigma\gamma(1-\alpha) - \gamma\theta(1-\sigma)},$$

$$D_{22} = \frac{\left\{\frac{\theta-\sigma(1-\alpha+\theta)}{\theta}(1-\tau)\gamma(1-\alpha) + \frac{[\gamma(1-\alpha)+1]^2}{\gamma\theta}\eta_{ss}\right\}\alpha\frac{y_{ss}}{k_{ss}}}{\frac{\sigma[1+\gamma(1-\alpha)]\eta_{ss}}{[1+\gamma(1-\alpha)]\eta_{ss}-\gamma(1-\alpha)(1-\tau)} + \sigma\gamma(1-\alpha) - \gamma\theta(1-\sigma)}.$$

Using matrix notations, we thus have:

$$\begin{pmatrix} \frac{d\ln k_t}{dt} \\ \frac{d\ln \eta_t}{dt} \end{pmatrix} = D \begin{pmatrix} \ln k_t - \ln k_{ss} \\ \ln \eta_t - \ln \eta_{ss} \end{pmatrix},$$

where

$$D = \begin{pmatrix} D_{11} & D_{12} \\ D_{21} & D_{22} \end{pmatrix}.$$

In what follows, we first show how to draw a phase diagram of the economy. Then, we characterize the short-run effects of a tighter environmental policy on c_t, η_t and k_t.

3.3.1. The phase diagram

To analyze how the economy converges to the steady state in a simple way, we draw the phase diagram in the (k ,η) space. For simplicity, we apply numerical methods. The benchmark value of parameters used to conduct the analysis is gathered in Table 3. We assume that the pollution tax can vary within [0%, 5%].[6] After computations, for any level of pollution tax within this range we have:

$$D_{11} < 0,\ D_{12} < 0,\ D_{21} < 0,\ D_{22} > 0, \tag{22}$$

[6] Brock and Taylor (2004) showed that abatement spending in the order of 1-2% of GDP seems to be the norm in OECD countries. In our paper, we allow abatement spending to vary within the range [0%, 5%] to cover more possibilities.

$$\left.\frac{d\eta_t}{dk_t}\right|_{\dot{k}_t = 0} = -\frac{D_{11}}{D_{12}} < 0 \text{ , and } \left.\frac{d\eta_t}{dk_t}\right|_{\dot{\eta}_t = 0} = -\frac{D_{21}}{D_{22}} > 0 \ . \tag{23}$$

To determine the stability properties of the model, we need to determine the sign of the eigenvalues (Λ_1, Λ_2) of matrix D. As the determinant of D is given by:

$$\det(D) = \Lambda_1\Lambda_2 = D_{11}D_{22} - D_{21}D_{12} < 0 \ , \tag{24}$$

the two eigenvalues have opposite signs. Hence, the unique steady state is saddle point stable.

We now characterize the slope of the saddle path. As the dynamic system is log-linearized, to compute the slope of the balanced growth path we can infer that:

$$\ln\eta_t - \ln\eta_{ss} = \Omega\left(\ln k_t - \ln k_{ss}\right) \ , \tag{25}$$

where Ω is the slope of the transitional path,

$$\Omega > 0 \ . \tag{26}$$

The exact value of Ω is computed as follows. Plugging (25) in (20) yields

$$\frac{d\ln k_t}{dt} = \left(D_{11} + \Omega D_{12}\right)\left(\ln k_t - \ln k_{ss}\right) \ . \tag{27}$$

Then, plugging (25) into (21), we get

$$\Omega\frac{d\ln k_t}{dt} = \left(D_{21} + \Omega D_{22}\right)\left(\ln k_t - \ln k_{ss}\right) \ . \tag{28}$$

Equations (27) and (28) imply that:

$$D_{11} + \Omega D_{22} = \frac{D_{21} + \Omega D_{22}}{\Omega} \ .$$

Solving this equation for Ω, we obtain

$$\Omega = \frac{D_{22} - D_{11} \pm \sqrt{\left(D_{11} - D_{22}\right)^2 + 4D_{12}D_{21}}}{2D_{12}} > 0 \ . \tag{29}$$

It will be important to keep this result in mind when we will turn to the construction of the phase diagram below, to the analysis of the effects of a tighter environmental policy in Section 3.3.2 and for the analysis of the static comparative in Section 3.4.

From (22), (23) and (29), the phase diagram can be drawn as in Figure 2:

Figure 2 here

The phase diagram displays the changes in physical capital per unit of efficient labor and the fraction of output devoted to healthcare services around the steady state. The upward-sloping curve corresponds to combinations of (k_t, η_t) for which $\dot{\eta}_t = 0$, whereas the downward-sloping curve corresponds to combinations of (k_t, η_t) for which $\dot{k}_t = 0$. The steady state of economy is depicted by the intersection of these two loci $\dot{\eta}_t = 0$ and $\dot{k}_t = 0$. Then, we must see how the variables behave outside the steady state. For equation (20), we see that k_t increases when η_t is lower than its steady-state value. Thus, arrows must point East on the area below the locus $\dot{k}_t = 0$. Similarly, k_t decreases when η_t is higher than its steady state value. The corresponding arrows must point West on the area above the locus $\dot{k}_t = 0$. We proceed in the same way for abatement spending. When k_t is lower than the value for which $\dot{\eta}_t = 0$, η_t increases. Thus, arrows must point North on the left hand side of the locus $\dot{\eta}_t = 0$. Finally, if k_t exceeds the value that yields $\dot{k}_t = 0$, η_t decreases. The corresponding arrows must point South on the right hand side of the locus $\dot{\eta}_t = 0$.

The direction of the arrows shows that the balanced growth path has a positive slope. Let us assume that the economy is initially on the balanced growth path. If the starting location is above the locus $\dot{\eta}_t = 0$ and below the locus $\dot{k}_t = 0$, both k_t and η_t increase over time to the steady state. Conversely, if the starting location is below the locus $\dot{\eta}_t = 0$ and above the locus $\dot{k}_t = 0$, both k_t and η_t decrease over time to the steady state.

3.3.2. The effects of a tighter environmental policy

Assuming that the government tightens the environmental policy, we now analyze the effects of this change on k_t, η_t and c_t in two cases: (i) the pollution-tax rate, before and after changing, is low (τ increases from 1% to 1.3%); and (ii) the pollution-tax rate is high (τ increases from 1.75% to 1.81%). Computations show that $\tau_{c,k}^{\max} = 1.746\%$ and $\tau^{w} = 1.813\%$. Following the definition of the proxy for the development level of a country mentioned in the welfare analysis, the first case would represent the developing countries and the second case would represent the developed countries. Given the benchmark value of the parameters in Table 3, we draw Figure 3 for the first case and Figure 4 for the second one.

Figure 3 and Figure 4 here

In each case, the two dashed lines denote the loci $\dot{\eta}_t = 0$ and $\dot{k}_t = 0$ for the initial tax rate. The intersection point of these two lines indicates the old steady state ($k_{ss}^{old}, \eta_{ss}^{old}$). Similarly, the solid lines denote the loci determined after the policy changes, and the intersection represent the new steady state ($k_{ss}^{new}, \eta_{ss}^{new}$). The balanced growth path is the dash-dotted line going through the steady state.

Let us assume that the amount of physical capital per unit of efficient labor at the time of the change is given by $k_t = k_{ss}^{old}$. As physical capital per unit of efficient labor is a predetermined variable, it cannot jump freely. By contrast, the fraction of output devoted to healthcare services, η_t, is a control variable and thus free to jump. In order to converge into the new steady state, η_t must be set on the new balanced growth path at the time of the change in the policy instruments. However, to know whether η_t jumps up or down, we must determine the position of the new balanced growth path: if it locates above the starting point of the economy, η_t must increase; conversely, if it locates below the starting point, η_t must decrease.

In Figure 3, the new balanced growth path locates below the old steady state and $k_{ss}^{old} < k_{ss}^{new}$. Hence, η_t must jump down at first, and then increases together with k_t to the new steady state. Equation (19) states that consumption, c_t, must jump down at the time of the change in the environmental policy, and then increases to the new steady state. The reason behind this result is following. At the time of the change, the tax burden reduces consumption and healthcare spending. However, because the government spends a low fraction of output on abatements, the productivity of abatements is high. The health benefit from the improvement of environmental quality increases labor productivity, thus, boosts growth. This allows individuals to spend more resource on consumption and healthcare services.

In Figure 4, the new balanced growth path also locates below the old steady state, but $k_{ss}^{old} > k_{ss}^{new}$. Hence, η_t must jump down at first, and then decreases together with k_t to the new steady state. Equation (19) states that c_t must jump down at the time of the change, and then decreases to the new steady state. That is, the economy must substitute consumption and healthcare spending for abatements in the short run. Moreover, the productivity of abatements is so small that an increase in abatement spending crowds out capital investments leading to a decline in growth. It leads to a decrease in consumption and healthcare spending in the long run.

3.4. Static comparative

In this section, we construct some static comparative to examine the short-run and long-run effects of an increase in the population growth rate, n, the growth rate of technical progress, g_A, and the rate of time preferences, ρ, on the variables η_t, c_t, and k_t.

Applying numerical methods and using the benchmark values given in Table 3, we find that (22) and (23) are still verified. Thereby, we can draw Figure 5, 6 and 7 which represent the economy before and after the changes.

Figure 5, 6, 7 here

The values of η_{ss}, c_{ss}, and k_{ss} before and after the change, are shown in Table 4, where a sign "-" indicates a negative change and correspondingly a sign "+" indicates a positive change.

Table 4: The effects of a change in n, g_A, and ρ on η_{ss}, c_{ss}, and k_{ss}.

	Base values (see Table 5)	n increases (from 1.25% to 1.75%)		g_A increases (from 0.02% to 0.025%)		ρ increases (from 0.04% to 0.045%)	
		Value	Change	Value	Change	Value	Change
η_{ss}	10.89%	10.86%	-	10.88%	-	10.91%	+
c_{ss}	0.0691	0.0662	-	0.0652	-	0.0676	-
k_{ss}	0.3178	0.2918	-	0.2745	-	0.2917	-

As shown in Table 4, at the time of the change in population growth, η_t jumps down on the new balanced growth path**.** Then, both k_t and η_t decrease continuously until the economy reaches the new steady state. Equation (19) shows that c_t jumps down when n changes, then it decreases to the new steady state. Moreover, in the long run, the value of η_{ss}, c_{ss}, and k_{ss} are lower. The explanation for this result is following. As efficient labor grows at rate $n+g_A$, the quantity of available capital must be shared among an increasing number of more skilled individuals (dilution effect). That is, when n increases, individuals have less resource for consumption, healthcare spendings and capital accumulation.

Second, if the growth rate of technical progress, g_A, increases from 0.02 to 0.025, Figure 5 shows that η_t jumps up on the new balanced growth path. It then decreases together with k_t until the economy reaches the new steady state. Equation (19) reveals that c_t jumps down when n changes, then it decreases to the new steady state. Finally, in the long run, the value of η_{ss}, c_{ss}, and k_{ss} are lower. The reason behind this result is similar to the previous case: there is a dilution effect caused by the increase in the growth rate of technical progress.

Third, if the rate of time preference, ρ, increases from 0.04 to 0.045, individuals put a higher weight on current consumption and health relative to the future. Figure 6 shows that η_t jumps up on the new balanced growth path. Then it decreases together with k_t until the economy reaches the new steady state. Equation (19) shows that c_t jumps up at first, and then it decreases. In the long run, c_{ss} and k_{ss} are higher but η_{ss} is lower. This is because individuals are willing to sacrifice future consumption and wealth in exchange for more current consumption and healthcare spending. Facing the decline in physical capital per unit of efficient labor in the long run, individuals increase healthcare spending.

4. Conclusion

This paper developed a growth model in which we set out the complexity of the relations among pollution, health and economic growth. In our model, health has a direct effect on individuals' welfare and is a proxy of their productivity. Health is improved by spending private resource on healthcare services, while it is reduced by pollution emissions coming from output production. These emissions can be reduced via abatement technologies funded by a pollution tax on output.

In such economy, we investigated the effects of this pollution tax on individuals' decisions with respect to consumption, healthcare spending and savings in both the short run and the long run. We found that an increase in the pollution tax reduces the long-run level of healthcare spending. This is because both elements are treated as imperfect substitutes as suggested by empirical evidences. We then showed the existence of three inverted U-shaped relationships reminiscent from the trade-offs we analyzed in this paper. They include the relationship between: (i) the pollution tax and the level of health; (ii) the pollution tax and consumption; (iii) the pollution tax and welfare. Finally, we analyzed the responses of the economy to a change in some structural parameters such as the growth rate of population, the growth rate of technical progress and the rate of time preference, with some numerical applications.

To keep the analysis simple and to focus on the key features of the problem, we presented a simple framework. Several extensions are possible. Future research could, for instance, consider an endogenous growth model to analyze the effects of pollution on long-run growth. Second, heterogeneity between individuals could also be introduced to study the effects of an environmental policy on economic development and inequality. Finally, empirical assessments would be interesting to see the relations among the variables in our model. These issues are on our agenda for future work.

5. References

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6. Appendix

6.1. Steady state

Differentiating (9) with respect to time yields: $(1-\sigma)g_A-\sigma\dot{c}_t/c_t+\gamma\theta(1-\sigma)\dot{\eta}_t/\eta_{tt}.$ $=\dot{\lambda}_t/\lambda$. Combining this with (11) we have:

$$\sigma\frac{\dot{c}_t}{c_t}-\gamma\theta(1-\sigma)\frac{\dot{\eta}_t}{\eta_t}=\alpha\left(1-\eta_t-\tau\right)\frac{y_t}{k_t}-\sigma g_A-\rho-n\ . \tag{30}$$

From (9) and (10), we obtain:

$$c_t=\frac{\left[1+\gamma(1-\alpha)\right]\eta_t-\gamma(1-\alpha)(1-\tau)}{\gamma\theta}B\left(k_t\right)^{\alpha}\left[\phi\left(\eta_t\right)^{\gamma}(\tau)^{\beta\chi}\right]^{1-\alpha}\ . \tag{31}$$

The results of this paper depend mostly on the conditions (6), (30) and (31). We note that $\dot{y}_t=\dot{k}_t=\dot{c}_t=\dot{h}_t=\dot{\eta}_t=0$ at steady state. Using (30) and (31), we have:

$$c_t=\frac{\left\{\left[1+\gamma(1-\alpha)\right]\eta_t-\gamma(1-\alpha)(1-\tau)\right\}\left(\sigma g_A+\rho+n\right)}{\gamma\theta\alpha}k_t\ . \tag{32}$$

Combining (15) and (32), we obtain the fraction of output devoted to healthcare services given by (12). Plugging (12) in (3), we get the level of health given by (13). Plugging (12) in (30), we have the level of physical capital per unit of efficient labor given by (14). Using (6) and (30), we get the relationship between consumption and physical capital per unit of efficient labor as shown by (15).

6.2. Pollution tax for maximum consumption, physical capital and health

Note that $\sigma>1$ (Barro and Sala-i-Martin, 2004). This condition is very useful in simplifying our computations. At first, we find the level of pollution tax that maximizes consumption and the level of health. Let us define $\tau_c^{\max}=agr\max c_{ss}$; and $\tau_k^{\max}=agr\max k_{ss}$. Direct inspection on (15) yields: $\partial k_{ss}/\partial\tau=\partial c_{ss}/\partial\tau$. Thus, if there is a level of tax that maximizes physical capital, then it maximizes consumption. We denote this tax level by $\tau_c^{\max}=\tau_k^{\max}=\tau_{c,k}^{\max}$. Differentiating (14) with respect to τ, we have:

$$\frac{\partial k_{ss}}{\partial\tau}=\frac{k_{ss}}{(1-\alpha)\left(1-\eta_{ss}-\tau\right)}\left[\begin{array}{c}\dfrac{(\gamma-\gamma\alpha)(1-\tau)-(\gamma-\alpha\gamma+1)\eta_{ss}}{\eta_{ss}}\dfrac{\partial\eta_{ss}}{\partial\tau}\\-\dfrac{(\beta\chi-\alpha\beta\chi+1)\tau-\beta\chi(1-\alpha)\left(1-\eta_{ss}\right)}{\tau}\end{array}\right] .(33)$$

Simple computations applied on (12) give: $\partial\eta_{ss}/\partial\tau=-\eta_{ss}/(1-\tau)<0$. Using this result with (12) to solve $\partial k_{ss}/\partial\tau=0$, we get:

$$\tau_{c,k}^{\max}=\frac{\beta\chi(1-\alpha)}{(\beta\chi+\gamma)(1-\alpha)+1}\ .$$

Since $c_t>0$, (32) reveals that $\eta_{ss}>\gamma(1-\alpha)(1-\tau)/[1+\gamma(1-\alpha)]$. From (33), we see that the term outside the square bracket is positive. Thus the sign of $\partial k_{ss}/\partial\tau$ depends on how high the value of the second term inside the square bracket is. After some manipulations we can show that: $\partial k_{ss}/\partial\tau>0$ if $\tau<\tau_{c,k}^{\max}$ and $\partial k_{ss}/\partial\tau<0$ if $\tau>\tau_{c,k}^{\max}$.

We now find the tax rate that maximizes the level of health. Differentiating (13) with respect to τ yields:

$$\frac{\partial h_{ss}}{\partial\tau}=\gamma\frac{h_{ss}}{\eta_{ss}}\frac{\partial\eta_{ss}}{\partial\tau}+\beta\chi\frac{h_{ss}}{\tau}\ .$$

We define $\tau_h^{\max}=agr\max h_{ss}$. Solving $\partial h_{ss}/\partial\tau=0$ and using $\partial\eta_{ss}/\partial\tau=-\eta_{ss}/(1-\tau)$, we obtain:

$$\tau_h^{\max}=\frac{\beta\chi}{\beta\chi+\gamma}\ .$$

Simple comparisons show that: $\partial h_{ss}/\partial\tau>0$ if $\tau<\tau_h^{\max}$, and $\partial h_{ss}/\partial\tau<0$ if $\tau>\tau_h^{\max}$.

6.3 Deriving the dynamic system

Taking the log-linearization (31), we obtain:

$$\ln c_t=\ln\left(\frac{[1+\gamma(1-\alpha)]e^{\ln\eta_t}-\gamma(1-\alpha)(1-\tau)}{\gamma\theta}\right)$$
$$+\ln B\left(\phi\tau^{\beta\chi}\right)^{1-\alpha}+\ln\left(k_t\right)^{\alpha}+\ln\left(\eta_t\right)^{\gamma(1-\alpha)}\ . \tag{34}$$

Then, differentiating both sides of (34) with respect to time around the steady state, we have:

$$\frac{d\ln c_t}{dt}=\left[\frac{[1+\gamma(1-\alpha)]\eta_{ss}}{[1+\gamma(1-\alpha)]\eta_{ss}-\gamma(1-\alpha)(1-\tau)}+\gamma(1-\alpha)\right]\frac{d\ln\eta_t}{dt}+\alpha\frac{d\ln k_t}{dt}\ . \tag{35}$$

Let us rewrite (30) as

$$\sigma \frac{d \ln c_t}{dt} - \gamma\theta(1-\sigma)\frac{d \ln \eta_t}{dt} = \alpha\left(1-\tau-\eta_t\right)\frac{B\left(k_t\right)^{\alpha}\left[\phi\left(\eta_t\right)^{\gamma}(\tau)^{\beta\chi}\right]^{1-\alpha}}{k_t} - \sigma g_A - \rho - n \ .$$

(36)

Combining (35) and (36) yields:

$$\left[\frac{\sigma[1+\gamma(1-\alpha)]\eta_{ss}}{[1+\gamma(1-\alpha)]\eta_{ss} - \gamma(1-\alpha)(1-\tau)} + \sigma\gamma(1-\alpha) - \gamma\theta(1-\sigma)\right]\frac{d \ln \eta_t}{dt} + \sigma\alpha \frac{d \ln k_t}{dt}$$
$$= \alpha\left(1-\tau-e^{\ln \eta_t}\right)B\left[\phi(\tau)^{\beta\chi}\right]^{1-\alpha} e^{(\alpha-1)\ln k_t + \gamma(1-\alpha)\ln \eta_t} - \sigma g_A - \rho - n \ .$$

(37)

Taking the first order of Taylor's expansion for the right hand side of (37), we have:

$$\left[\frac{\sigma[1+\gamma(1-\alpha)]\eta_{ss}}{[1+\gamma(1-\alpha)]\eta_{ss} - \gamma(1-\alpha)(1-\tau)} + \sigma\gamma(1-\alpha) - \gamma\theta(1-\sigma)\right]\frac{d \ln \eta_t}{dt} + \sigma\alpha \frac{d \ln k_t}{dt}$$
$$= \alpha\left(1-\tau-\eta_{ss}\right)(\alpha-1)\frac{y_{ss}}{k_{ss}}\left(\ln k_t - \ln k_{ss}\right) + \left[(1-\tau-\eta_{ss})\gamma(1-\alpha) - \eta_{ss}\right]\alpha\frac{y_{ss}}{k_{ss}}\left(\ln \eta_t - \ln \eta_{ss}\right) \ .$$

(38)

Dividing both sides of (6) by k_t, and then taking the first order of Taylor's expansion for the right hand side, we obtain:

$$\frac{d \ln k_t}{dt} = \left[\frac{1+\gamma(1-\alpha+\theta)}{\gamma\theta}\eta_{ss} - \frac{(1-\alpha+\theta)(1-\tau)}{\theta}\right](1-\alpha)\frac{y_{ss}}{k_{ss}}\left(\ln k_t - \ln k_{ss}\right)$$
$$+\left[\begin{array}{l}\dfrac{(1-\alpha+\theta)(1-\tau)\gamma(1-\alpha)}{\theta} \\ -\dfrac{\left[1+\gamma(1-\alpha+\theta)\right]\left[\gamma(1-\alpha)+1\right]}{\gamma\theta}\eta_{ss}\end{array}\right]\frac{y_{ss}}{k_{ss}}\left(\ln \eta_t - \ln \eta_{ss}\right) \ . \qquad (39)$$

Plugging (39) into (38) yields:

$$\frac{d\ln\eta_t}{dt}=\frac{\left[\left(1-\sigma\frac{1+\gamma(1-\alpha+\theta)}{\gamma\theta}\right)\eta_{ss}-1+\tau+\sigma\frac{(1-\alpha+\theta)(1-\tau)}{\theta}\right]\alpha(1-\alpha)\frac{y_{ss}}{k_{ss}}}{\frac{\sigma[1+\gamma(1-\alpha)]\eta_{ss}}{[1+\gamma(1-\alpha)]\eta_{ss}-\gamma(1-\alpha)(1-\tau)}+\sigma\gamma(1-\alpha)-\gamma\theta(1-\sigma)}\left(\ln k_t-\ln k_{ss}\right)$$

$$+\frac{\left\{\frac{\theta-\sigma(1-\alpha+\theta)}{\theta}(1-\tau)\gamma(1-\alpha)+\frac{[\gamma(1-\alpha)+1]^2}{\gamma\theta}\eta_{ss}\right\}\alpha\frac{y_{ss}}{k_{ss}}}{\frac{\sigma[1+\gamma(1-\alpha)]\eta_{ss}}{[1+\gamma(1-\alpha)]\eta_{ss}-\gamma(1-\alpha)(1-\tau)}+\sigma\gamma(1-\alpha)-\gamma\theta(1-\sigma)}\left(\ln\eta_t-\ln\eta_{ss}\right)\ .$$

(40)

Equations (39) and (40) are the equations (20) and (21) respectively.

Table 3: Baseline parameter values

Description	Parameter	Base value	Range	Source
Population growth rate	n	0.0125	[0,0.025]	OECD in figures 2009
Growth rate of technical progress	g_A	0.02	[0.01,0.03]	Barro and Sala-i-Martin (2004)
Rate of time preferences	ρ	0.04	[0.02,0.07]	
Weight of health in utility	θ	0.2		
Capital share in the output sector	α	0.35		Brock and Taylor (2005)
Inverse elasticity of pollution with respect to abatements	β	0.3		Pautrel (2008)
Elasticity of health with respect to healthcare spending	γ	0.15		
Elasticity of health with respect to environmental quality	χ	0.1		
Inverse of the elasticity of substitution	σ	1.75		Barro and Sala-i-Martin (2004)
Constant productivity parameter of the health sector	ϕ	0.3		
Constant productivity parameter of the output sector	B	0.4		
Pollution tax	τ	0.01	[0.01,0.02]	Brock and Taylor (2005)

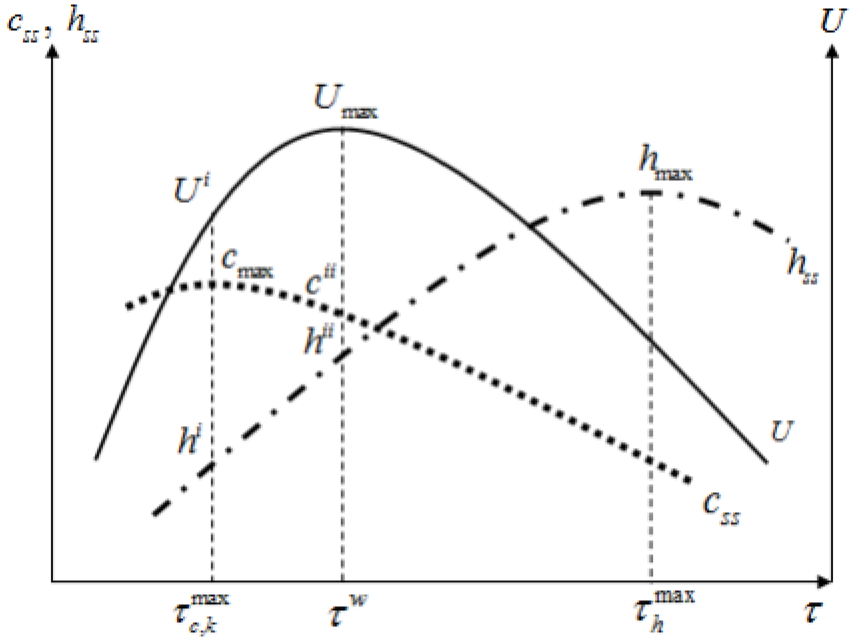

Figure 1: The three inverted-U relationship.

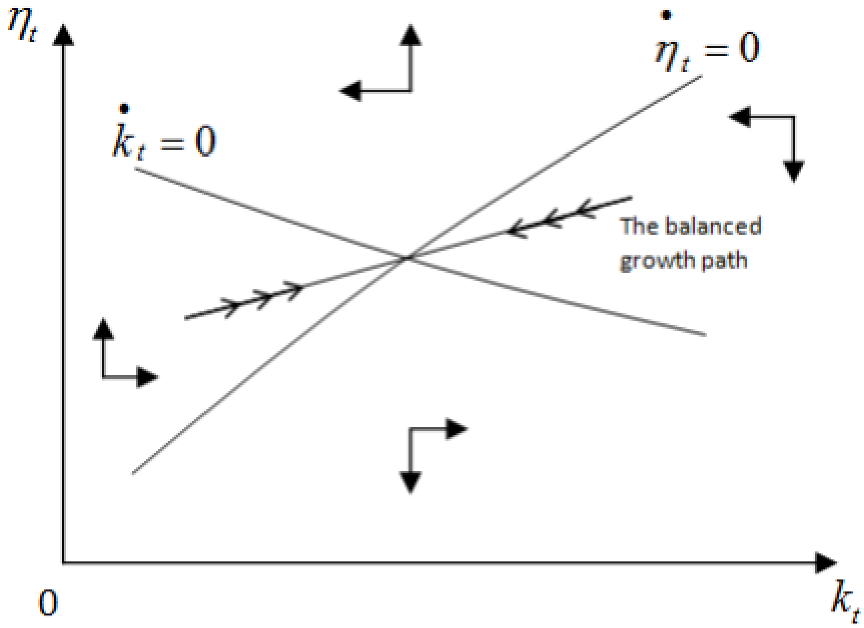

Figure 2: The phase diagram.

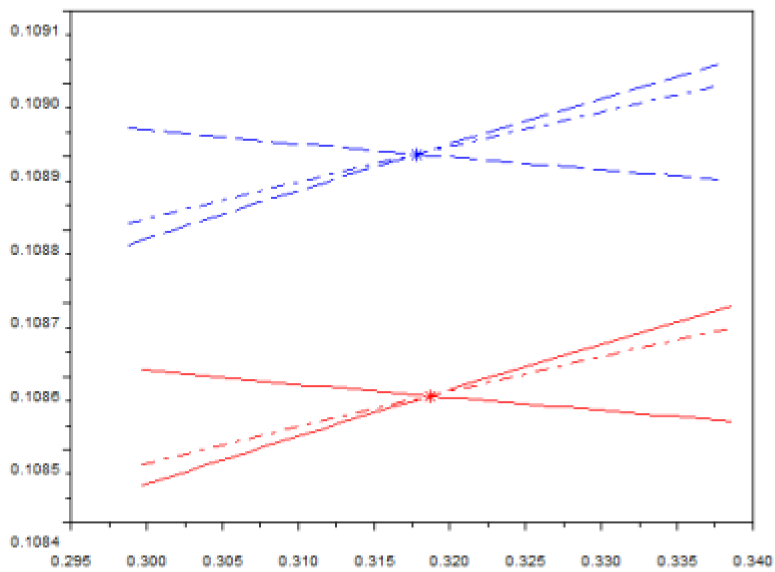

Figure 3: Pollution tax increases from 1% to 3%.

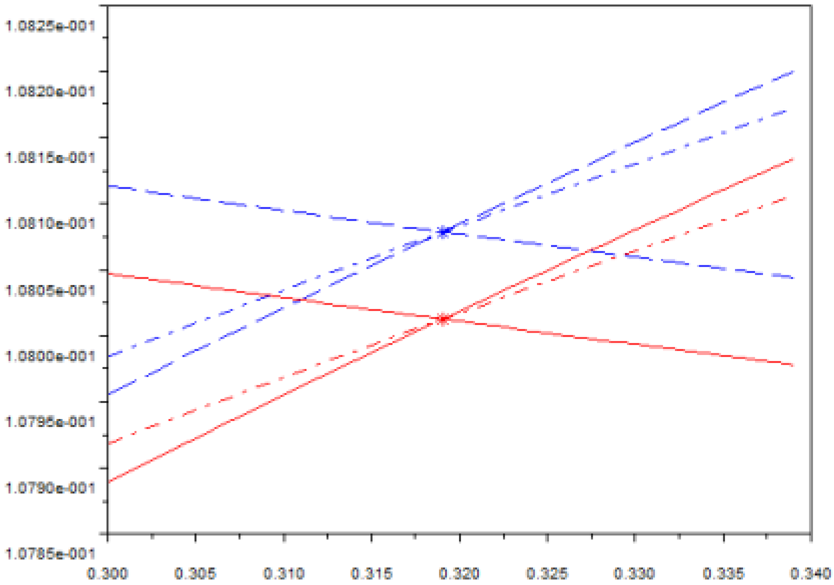

Figure 4: Pollution tax increases from 1.75% to 1.81%.

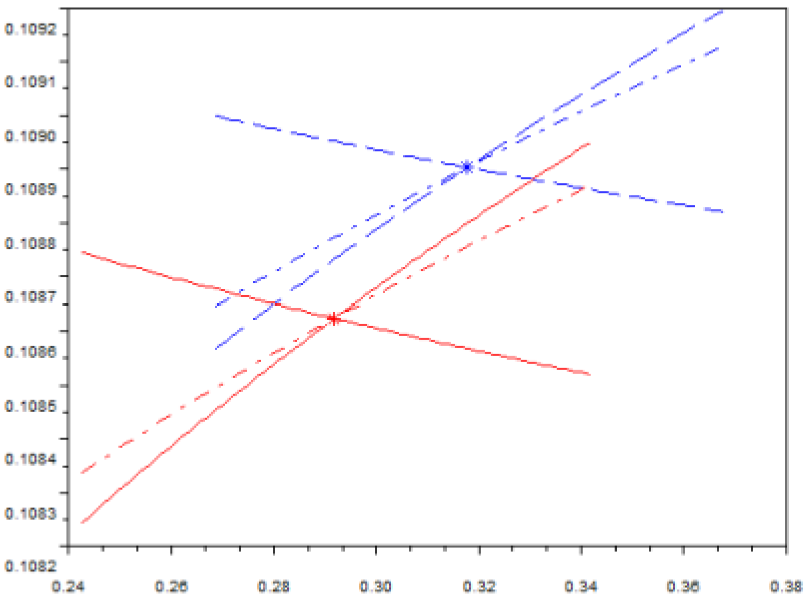

Figure 5: The population growth rate increases.

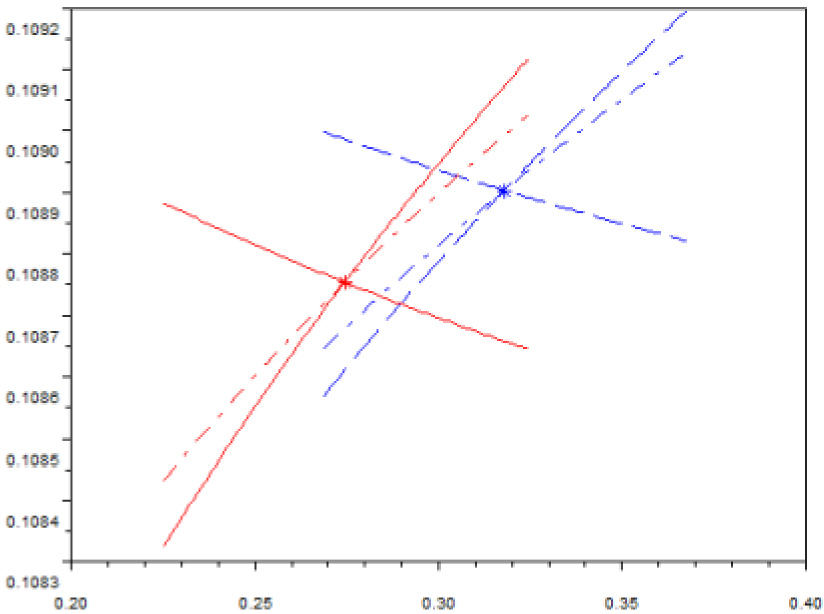

Figure 6: The growth rate of technical progress increases.

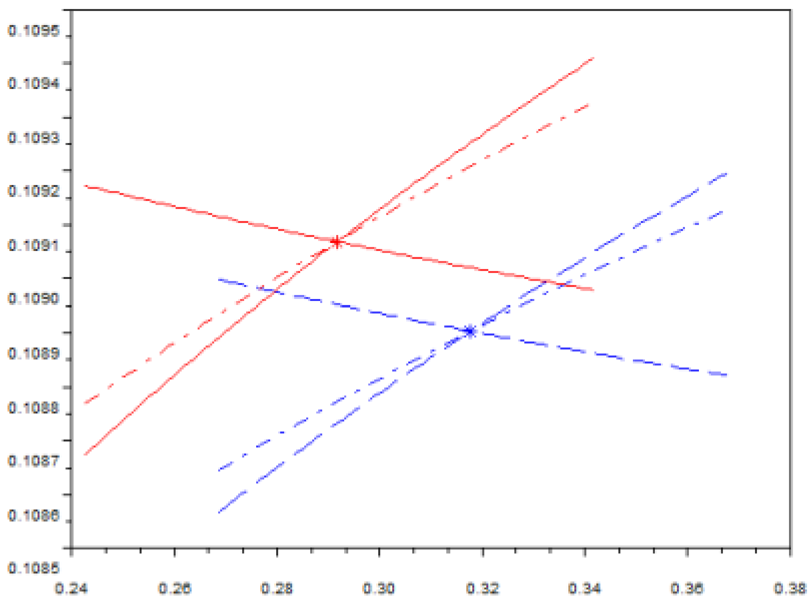

Figure 7: The time preferences increases.